Chapter 5

Loop Antennas

Loop antennas feature:

1- Simplicity, low cost and versatility .

2- They may have various shapes: circular, triangular, square, elliptical, etc .

3- They are widely used in communication links up to the microwave bands (up to 3 GHz). They are also used as electromagnetic (EM) field probes in the microwave bands.

Electrically small loops are widely used as compact transmitting and receiving antennas in the low MHz range (3 MHz to 30 MHz, or wavelengths of about 10 m to 100 m).

Single Circular Loop

Fig. 5.1(a)

Geometry for Circular Loop

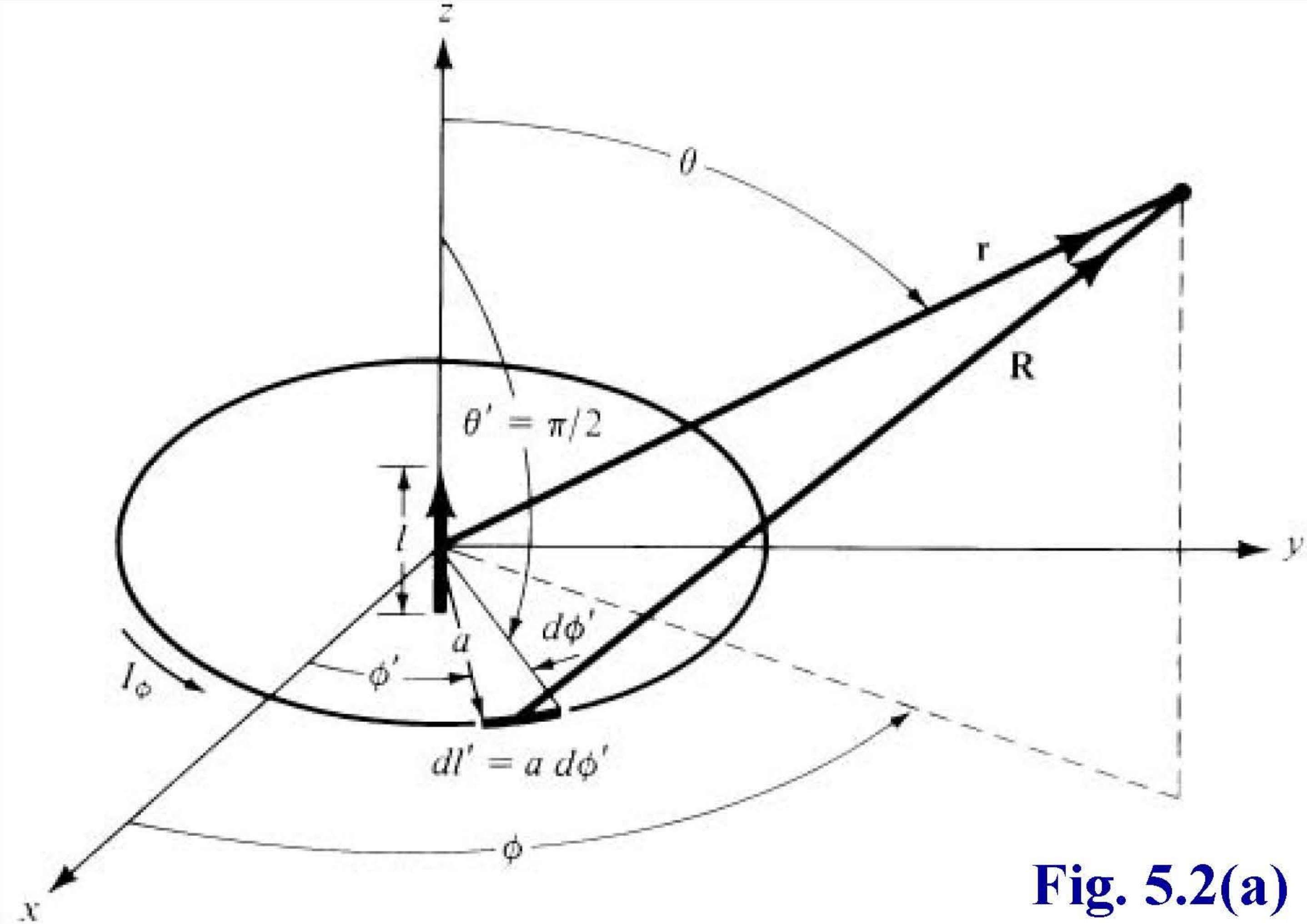


Fig. 5.2(a)

From Chapter 3:

$$\underline{A} = \frac{\mu}{4\pi} \int_C \underline{I}_e(x', y' z') \frac{e^{-jkR}}{R} d\ell' \quad (5\text{-}2)$$

R is the distance from any point on the current (source) $\underline{I}_e$ to the observation point.

$$\underline{A} \simeq \hat{a}_\phi A_\phi = \frac{a^2 \mu I_o}{4} e^{-jkr} \left(\frac{jk}{r} + \frac{1}{r^2} \right) e^{-jkr} \sin\theta \Rightarrow H = \frac{1}{\mu} \nabla \times \underline{A} \quad \Rightarrow$$

$$H_r = j \frac{k a^2 I_o \cos\theta}{2r^2} \left[1 + \frac{1}{jkr} \right] e^{-jkr} \qquad (5\text{-}18a)$$

$$H_\theta = -\frac{(ka)^2 I_o \sin\theta}{4r} \left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2} \right] e^{-jkr} \qquad (5\text{-}18b)$$

$$H_\phi = 0 \qquad (5\text{-}18c)$$

$$\underline{E}_A = \frac{\nabla \times \underline{H}_A}{j\omega\varepsilon}$$

$$E_r = E_\theta = 0 \qquad (5\text{-}19a)$$

$$E_\phi = \eta \frac{(ka)^2 I_o \sin\theta}{4r} \left[1 + \frac{1}{jkr} \right] e^{-jkr} \qquad (5\text{-}19b)$$

Small Loop

$$E_r = E_\theta = H_\phi = 0 \tag{5-19a}$$

$$E_\phi = \eta \frac{(ka)^2 I_o \sin\theta}{4r}\left[1 + \frac{1}{jkr}\right]e^{-jkr} \tag{5-19b}$$

$$H_r = j\frac{ka^2 I_o \cos\theta}{2r^2}\left[1 + \frac{1}{jkr}\right]e^{-jkr} \tag{5-18a}$$

$$H_\theta = -\frac{(ka)^2 I_o \sin\theta}{4r}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right]e^{-jkr} \tag{5-18b}$$

Infinitesimal dipole

$$H_r = H_\theta = 0 \quad \text{(4-8a)}$$

$$H_\phi = j\frac{kI_e\ell}{4\pi r}\sin\theta\left[1+\frac{1}{jkr}\right]e^{-jkr} \quad \text{(4-8b)}$$

$$E_r = \eta\frac{I_e\ell\cos\theta}{2\pi r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$E_\theta = j\eta\frac{kI_e\ell\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]e^{-jkr}$$

$$E_\phi = 0$$

Table 3.2 DUAL QUANTITIES FOR ELECTRIC (**J**) AND MAGNETIC (**M**) CURRENT SOURCES

Electric Sources ($\mathbf{J} \neq 0, \mathbf{M} = 0$)	Magnetic Sources ($\mathbf{J} = 0, \mathbf{M} \neq 0$)
$\mathbf{E}_A$	$\mathbf{H}_F$
$\mathbf{H}_A$	$-\mathbf{E}_F$
$\mathbf{J}$	$\mathbf{M}$
$\mathbf{A}$	$\mathbf{F}$
ϵ	μ
μ	ϵ
k	k
η	$1/\eta$
$1/\eta$	η

Small Loop: Magnetic Dipole

$$E_r = E_\theta = H_\phi = 0 \qquad \text{(5-20a)}$$

$$E_\phi = j\frac{kI_m\ell\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr} \qquad \text{(5-20b)}$$

$$H_r = \frac{I_m\ell\cos\theta}{2\pi\eta r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr} \qquad \text{(5-20c)}$$

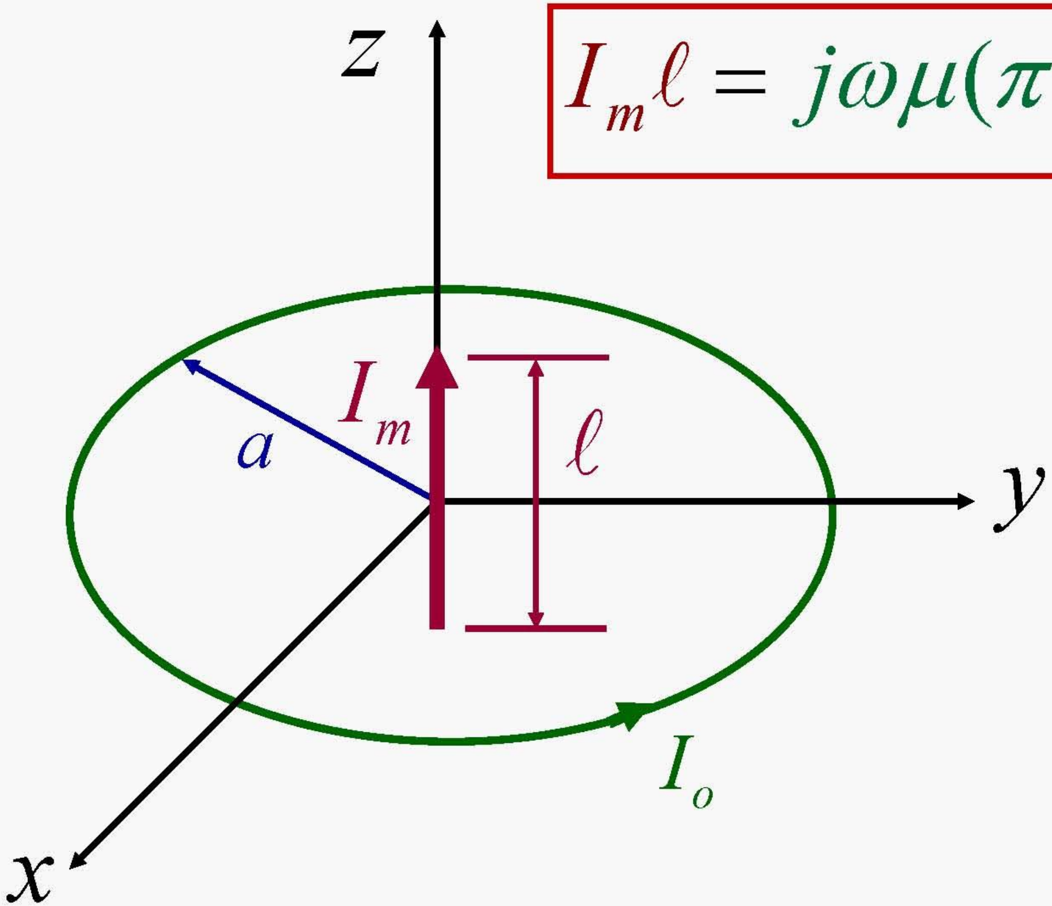
$$H_\theta = j\frac{kI_m\ell\sin\theta}{4\pi\eta r}\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]e^{-jkr} \qquad \text{(5-20d)}$$

Magnetic Dipole Equivalent:

$$I_m\ell = j\omega\mu I_o S,\ S = \pi a^2 \qquad \text{(5-21)}$$

Small Loop/Magnetic Dipole

$$I_m \ell = j\omega\mu(\pi a^2) I_o$$



Radiation Resistance

$$P_{rad} = \eta\left(\frac{\pi}{12}\right)(ka)^4 |I_o|^2 = \frac{1}{2}|I_o|^2 R_r$$

$$R_r = \eta\left(\frac{\pi}{6}\right)(ka)^4 = \eta\left(\frac{2\pi}{3}\right)\left(\frac{kS}{\lambda}\right)^2 = 20\pi^2\left(\frac{C}{\lambda}\right)^4 \quad (5\text{-}24)$$

$$S = \pi a^2$$

$$C = 2\pi a$$

1-Turn:

$$R_r = 20\pi^2 \left(\frac{C}{\lambda} \right)^4 \qquad (5\text{-}24)$$

N-Turns:

$$R_r \cong 20\pi^2 \left(\frac{C}{\lambda} \right)^4 N^2 \qquad (5\text{-}24a)$$

N-Turn Circular Loop

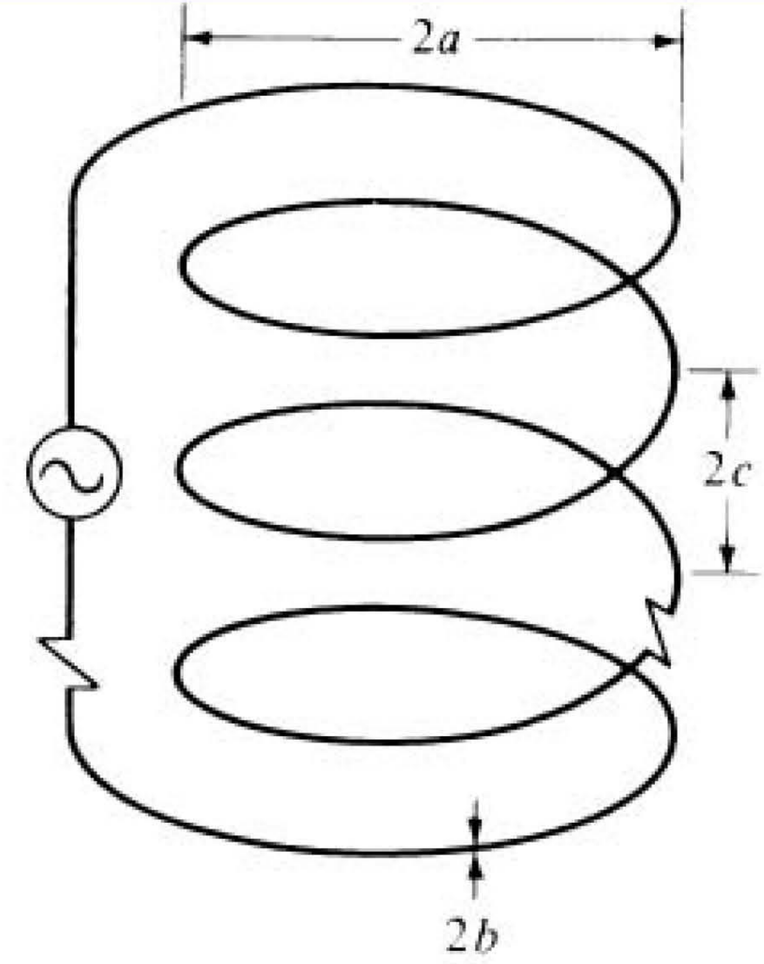


Fig. 5.3(a)

Proximity Effect of Turns

$$R_{ohmic} = \frac{Na}{b} R_s \left(\frac{R_p}{R_o} + 1 \right) \qquad (5\text{-}25)$$

$R_s = \sqrt{\dfrac{\omega \mu_o}{2\sigma}}$ = surface resistance of conductor

R_p = ohmic resistance due to proximity

$R_o = \dfrac{NR_S}{2\pi b}$ = ohmic skin effect resistance per unit length

Ohmic Resistance Due To Proximity

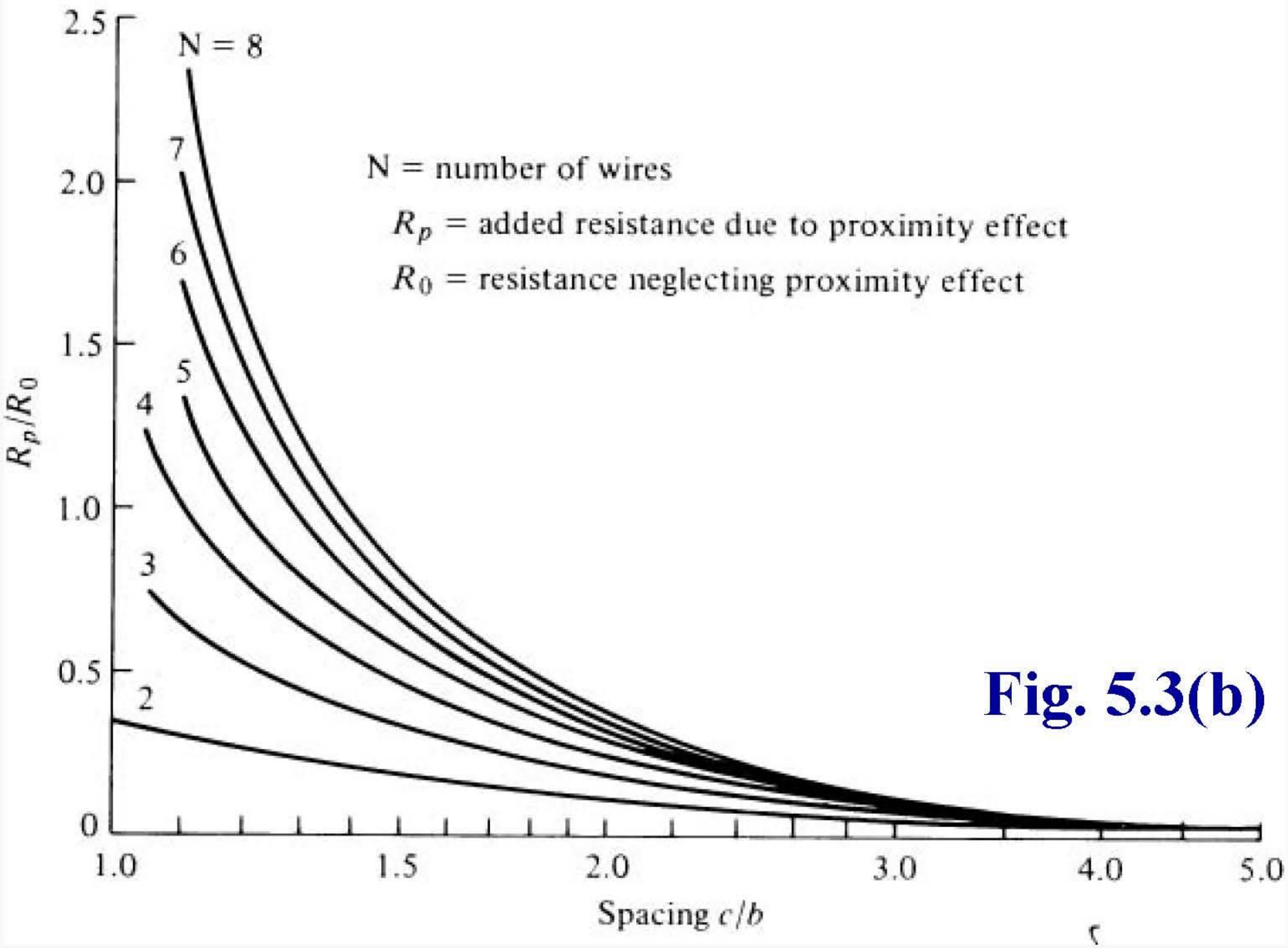


Fig. 5.3(b)

Far-Field

Number of turns
N=1

$$E_r = E_\theta = 0$$

$$E_\phi \cong \eta \frac{(ka)^2 I_o e^{-jkr}}{4r} \sin\theta$$

$$H_r \cong j \frac{ka^2 I_o e^{-jkr}}{2r^2} \cos\theta \cong 0$$

$$H_\theta \cong -\frac{(ka)^2 I_o e^{-jkr}}{4r} \sin\theta$$

$$H_\phi = 0$$

$$H_\theta = -\frac{E_\phi}{\eta}$$

$$D_o = 4\pi \frac{U_{\max}}{P_{rad}} = \frac{3}{2}$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_o = \frac{3\lambda^2}{8\pi}$$

Small Loop

Number of turns
N=1

$$R_r = 20\pi^2 \left(\frac{C}{\lambda}\right)^4 \quad (5\text{-}24)$$

$$D_o = \frac{3}{2} \quad (5\text{-}31)$$

$$A_{em} = \frac{3\lambda^2}{8\pi} \quad (5\text{-}32)$$

Elevation Plane Amplitude Patterns for a Circular Loop

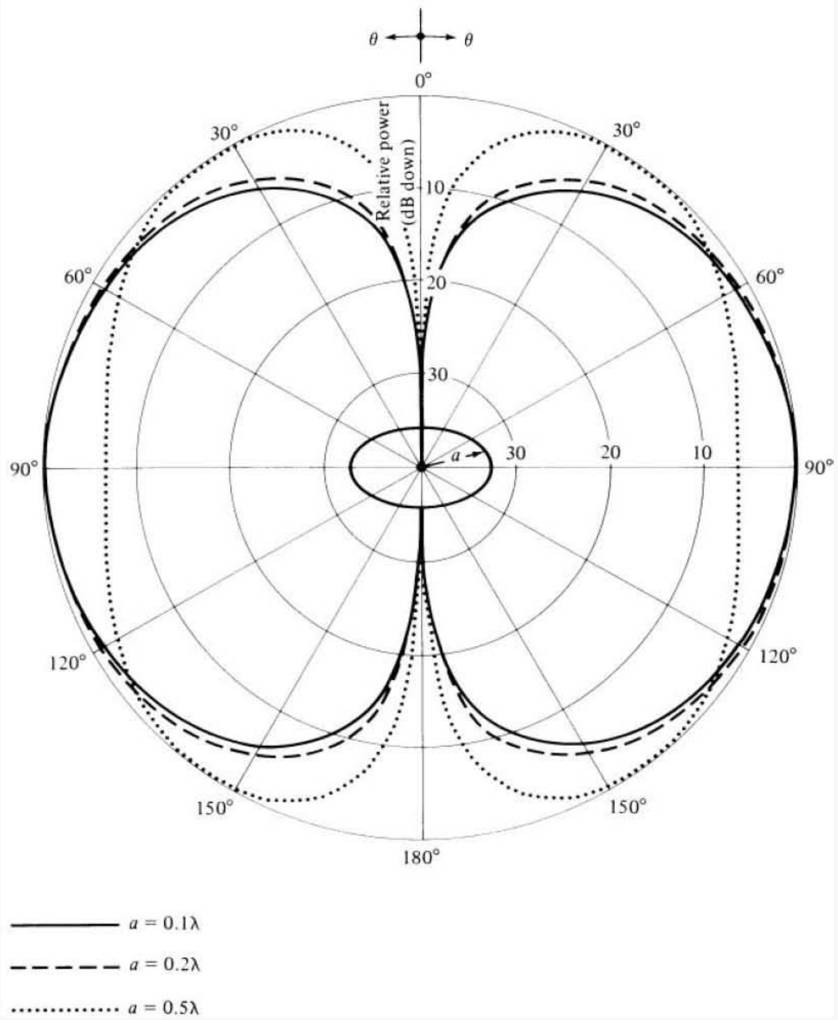


Fig. 5.7

3-D Pattern of Circular Loop with Uniform Current

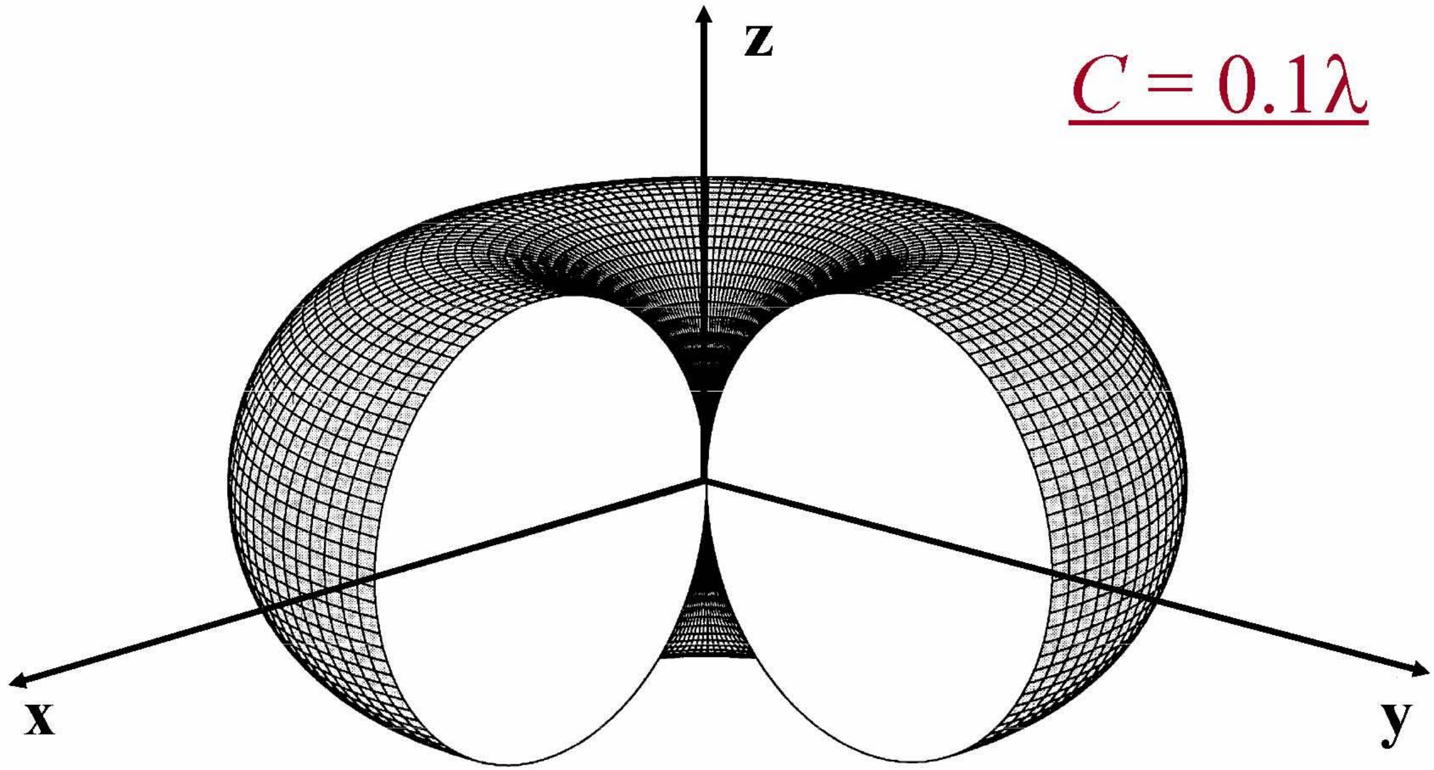
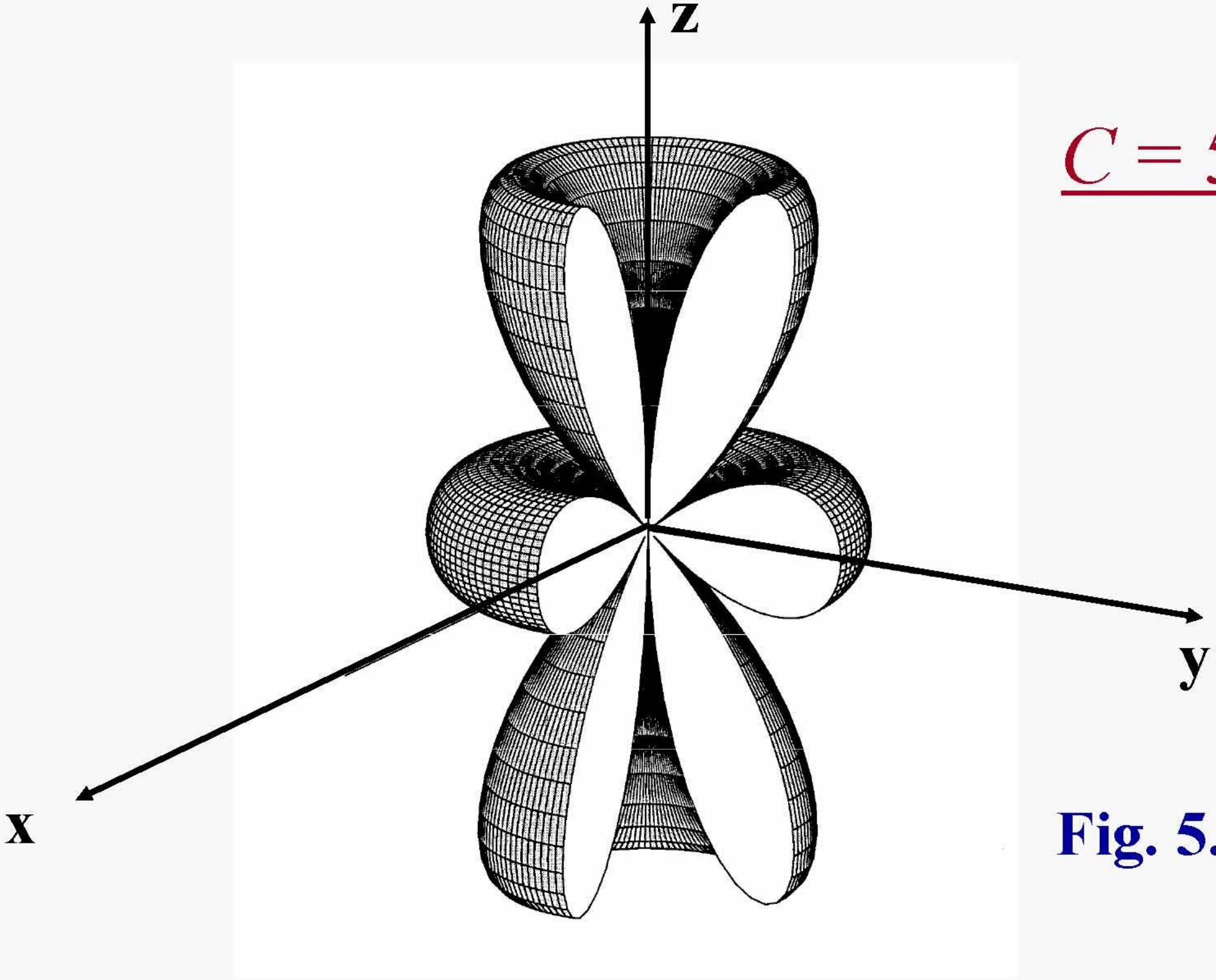


Fig. 5.8(a)

3-D Pattern of Circular Loop with Uniform Current



$C = 5\lambda$

Fig. 5.8(b)

Circuit Equivalent of Loop

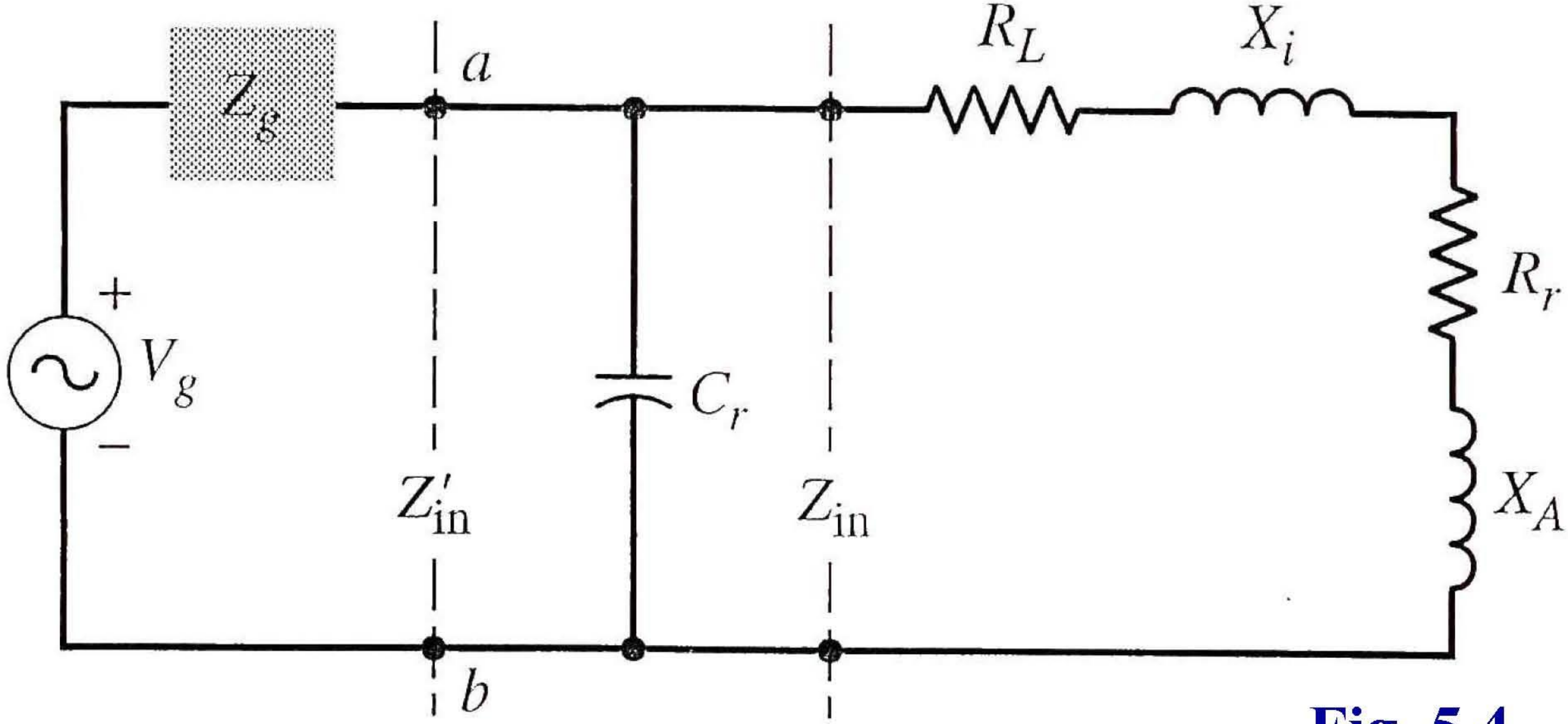


Fig. 5.4

Equivalent Circuit

A. Transmitting Mode

$$Z_{in} = R_{in} + jX_{in} = (R_r + R_L) + j(X_A + X_i) \quad (5\text{-}33)$$

R_r = radiation resistance as given by (5-24)

R_L = loss resistance of loop conductor

X_A = inductive reactance of loop antenna = ωL_A

X_i = reactance of loop conductor = ωL_i

$$R_{in} = R_r + R_L$$

$$X_{in} = X_A + X_i$$

$$Y_{in} = G_{in} + jB_{in} = \frac{1}{Z_{in}} = \frac{1}{R_{in} + jX_{in}} \quad (5\text{-}34)$$

$$G_{in} = \frac{R_{in}}{R_{in}^2 + X_{in}^2} \quad (5\text{-}34a)$$

$$B_{in} = -\frac{X_{in}^{2}}{R_{in}^2 + X_{in}^2} \quad (5\text{-}34b)$$

Choose C_r to eliminate B_{in}:

$$\omega C_r = 2\pi f C_r = B_r = -B_{in} = \frac{X_{in}^2}{R_{in}^2 + X_{in}^2}$$

$$C_r = \frac{1}{2\pi f} \cdot \frac{X_{in}^2}{R_{in}^2 + X_{in}^2} \quad (5\text{-}35)$$

At resonance:

$$Z'_{in} = R'_{in} = \frac{1}{G_{in}} = \frac{R_{in}^2 + X_{in}^2}{R_{in}} = R_{in} + \frac{X_{in}^2}{R_{in}} \quad (5\text{-}36)$$

$$R_{in} = R_r + R_L$$

$$X_{in} = X_A + X_i = \omega(L_A + L_i)$$

Inductances

Circular *(radius a, wire radius b)*

$$L_A = \mu_o a\left[\ell n\left(\frac{8a}{b}\right) - 2\right] \qquad (5\text{-}37a)$$

$$L_i = \frac{a}{\omega b}\sqrt{\frac{\omega\mu_o}{2\sigma}} \qquad (5\text{-}38)$$

Square *(sides a, wire radius b)*

$$L_A = 2\mu_o \frac{a}{b}\left[\ell n\left(\frac{a}{b}\right) - 0.774\right] \qquad (5\text{-}37b)$$

$$L_i = \frac{2a}{\omega\pi b}\sqrt{\frac{\omega\mu_o}{2\sigma}} \qquad (5\text{-}38)$$

Large Loop ($a > \lambda / 2$)

$$R_r = 60\pi^2 \left(\frac{C}{\lambda} \right) \quad \text{(5-63a)}$$

$$D_o = 0.677 \left(\frac{C}{\lambda} \right) \quad \text{(5-63b)}$$

$$A_{em} = 5.39 \times 10^{-2} \lambda C \quad \text{(5-63c)}$$

Ferrite Loop

$$\frac{R_f}{R_r} = (\mu_{cer})^2 \qquad (5\text{-}72)$$

R_f = radiation resistance of ferrite loop

R_r = radiation resistance of air core loop

μ_{cer} = relative effective permeability of ferrite core

$$R_f = 20\pi^2\left(\frac{C}{\lambda}\right)^4\left(\frac{\mu_{ce}}{\mu_o}\right)^2 = 20\pi^2\left(\frac{C}{\lambda}\right)^4\mu_{cer}^2 \quad \text{(1-Turn)} \qquad (5\text{-}73)$$

$$R_f = 20\pi^2\left(\frac{C}{\lambda}\right)^4\left(\frac{\mu_{ce}}{\mu_o}\right)^2 N^2 = 20\pi^2\left(\frac{C}{\lambda}\right)^4\mu_{cer}^2 N^2 \quad \text{(N-Turns)} \qquad (5\text{-}74)$$

The small loop antennas have the following properties:

1- high radiation resistance provided multi-turn ferrite - core constructions are use.

2- high losses, therefore, low radiation efficiency.

3- Simple construction, small size and weight.

Small loops are usually not used as transmitting antennas due to their low efficiency. However, they are much preferred as receiving antennas in AM radio- receivers because of their high signal-to-noise ratio (they can be easily tuned to form a very high-Q resonant circuit).